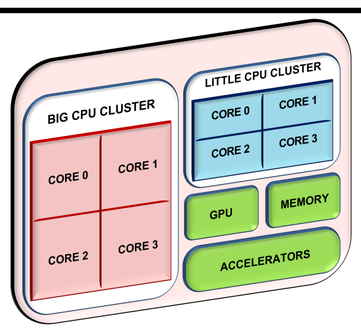
Power-Temperature Stability and Safety Analysis for Multiprocessor Systems

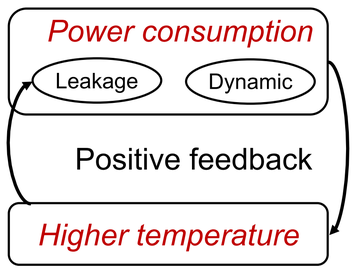
Ganapati Bhat, Umit Y. Ogras
Arizona State University

Suat Gumussoy
IEEE Member

Motivation

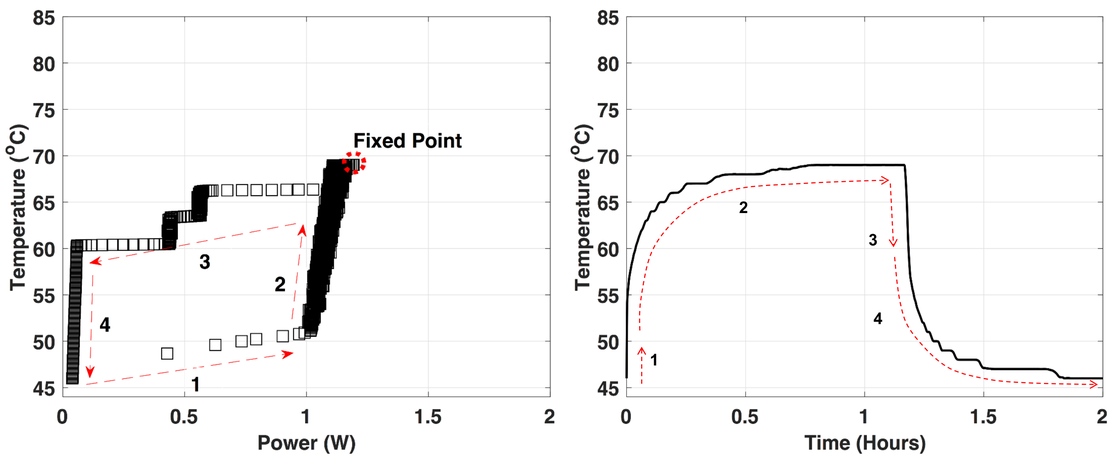
- Modern mobile platforms integrate multiple CPUs, GPUs and accelerators
- Power consumption and resulting heat dissipation is a major problem for mobile platforms [1]
  - High temperature affects user experience and reliability
  - Power and temperature form a positive feedback system
- Uncontrolled temperature can also cause safety risks





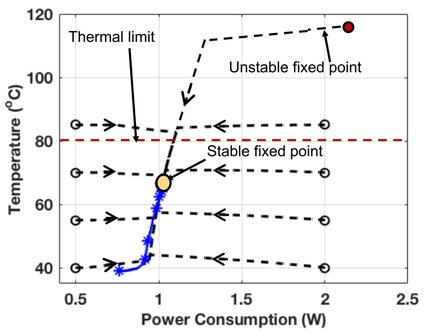
Fixed Point Illustration

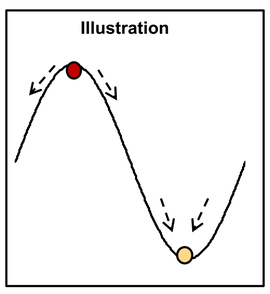
- A complete heat-up/cool-down cycle on Exynos 5422 SoC



Our Contributions

- **This work*:**
  1. Derives the necessary and sufficient conditions for the existence of the fixed point
  2. Proves the stability of the fixed point(s) and region of convergence
  3. Finds the maximum dynamic power consumption to guarantee a thermally safe temperature
  4. Is validated empirically on 8-core big.LITTLE platform





* The details can be found in [2]

Thermal and Power models

- **Temperature at a future time step can be written as**

$$\begin{bmatrix} T_1[k+1] \\ T_2[k+1] \\ \vdots \\ T_N[k+1] \end{bmatrix}_{N\times 1} = A_{N\times N}\begin{bmatrix} T_1[k] \\ T_2[k] \\ \vdots \\ T_N[k] \end{bmatrix}_{N\times 1} + B_{N\times M}\begin{bmatrix} P_1[k] \\ P_2[k] \\ \vdots \\ P_M[k] \end{bmatrix}_{M\times 1}$$

  - A models the effect of temperature in time step k in time step $k+1$
  - B models the effect of each power source on the temperature

- **Power consumption can be expressed as the sum of dynamic and temperature dependent leakage**

$$P_i = C_{sw,i}V_i^2 f_i + I_{g,i}V_i + V_i\kappa_{1,i}T_i^2 e^{\frac{\kappa_{2,i}}{T_i}}$$

$$P_i = P_{C,i} + V_i\kappa_{1,i}T_i^2 e^{\frac{\kappa_{2,i}}{T_i}}$$

where $C_{sw,i}$ is the switching capacitance, V_i is the voltage, f_i is the frequency, $I_{g,i}$ is the gate leakage current and $\kappa_{1,i}, \kappa_{2,i}$ are leakage power parameters for the ith resource $P_{C,i}$ is the temperature-independent component of power consumption

Fixed Point Function

- **Nonlinear MIMO system due to exponential terms in $P_i[k]$**
  - No closed-form solutions for this nonlinear system
- **Solution approach**
  - Reduce to SISO model to find an initial estimate of fixed points
  - Then, use Newton's method to solve the MIMO system
- **At steady state $(k \to \infty)$ we have**

$$T = aT + bP$$

where $0 < a < 1$ and $b > 0$ are parameters of reduced order SISO system

- **Using change of variables we can write,**

$$\beta\tilde{T}(1-\alpha\tilde{T}) = e^{-\tilde{T}}$$

- **Taking logarithm on both sides,**

$$\mathcal{F}(\tilde{T}) = \ln\beta + \ln\tilde{T} + \ln(1-\alpha\tilde{T}) + \tilde{T} = 0$$

Fixed Point Analysis

- **Lemma 1: $\mathcal{F}(\tilde{T})$ satisfies the following properties**
  - $\mathcal{F}(\tilde{T})$ is concave in the interval $\tilde{T} \in (0, \frac{1}{\alpha})$
  - $\mathcal{F}(\tilde{T})$ has a unique maxima at $\tilde{T}_m$ given by

$$\tilde{T}_m = \frac{1}{2\alpha} - 1 + \sqrt{\frac{1}{4\alpha^2} + 1}$$

  - $\mathcal{F}(\tilde{T})$ is increasing in $(0, \tilde{T}_m)$ and decreasing in $(\tilde{T}_m, \frac{1}{\alpha})$
- **Lemma 2: Value of $\tilde{T}$ in the temperature iteration increases when $\mathcal{F}(\tilde{T}) < 0$, and decreases when $\mathcal{F}(\tilde{T}) > 0$**
- **Theorem 2: Stability of fixed points is as follows**
  - When $\mathcal{F}(\tilde{T})$ has no solution, the temperature iteration diverges, i.e, $\tilde{T} \to 0$ $(T \to \infty)$
  - Where there are two fixed points, $\tilde{T}_u \in (0, \tilde{T}_m)$ is unstable and $\tilde{T}_s \in (\tilde{T}_m, 1/\alpha)$ is stable.

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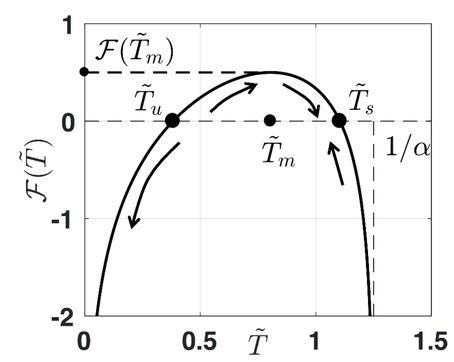


Temperature to Power Constraint

1. Calculate the auxiliary temperature as

$$\tilde{T}^* = -\frac{\kappa_2}{T^*}$$

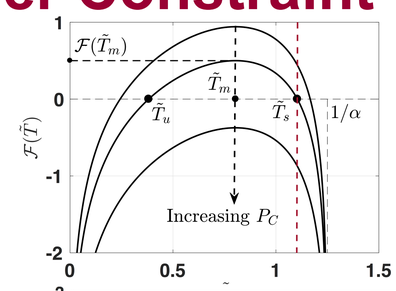
2. Given $\tilde{T}^*$, find α^* as

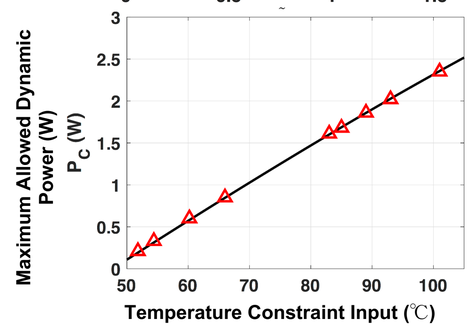
$$\alpha^* = \frac{1}{\tilde{T}^*}\left(1 - \frac{e^{-\tilde{T}^*}}{\beta\tilde{T}^*}\right)$$

3. Finally, using α^* we find P_C^* as

$$P_C^* = \frac{a-1}{b}\alpha^*\kappa_2$$

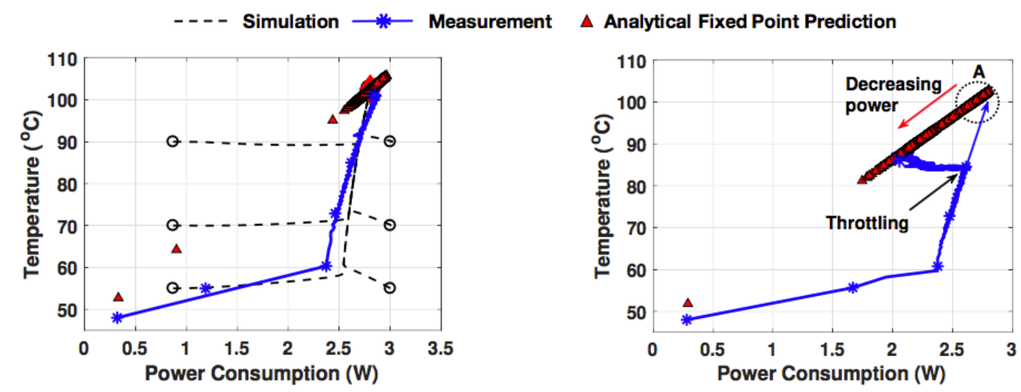
- **Maximum allowed power consumption for various temperature constraints**
  - Power constraint increases linearly with temperature constraint



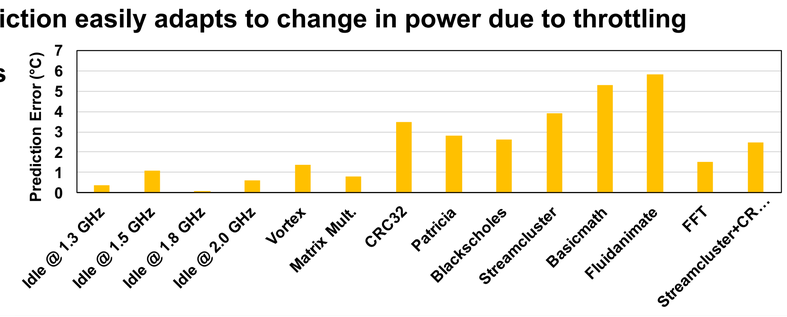


Experimental Results

- Time granularity of predictions



- **On the right side, the prediction easily adapts to change in power due to throttling**
- **Average prediction error is only 2.3 °C**
- **Worst case error is 5.8 °C**
  - When the temperature fixed point is ~100 °C, we did not run the experiment long enough to reach it



Conclusions

- **Theoretical stability analysis for power-temperature dynamics**
  - **Existence:** Derived sufficient and necessary conditions
  - **Stability:** Found region of stability
  - **Safety:** Derived maximum allowed dynamic power
- **Our technique takes 75.2 µs when integrated into Android OS**
- **We validated our approach empirically on the Odroid-XU3 board**

References

[1] Gaurav Singla, et al., "Predictive Dynamic Thermal and Power Management for Heterogeneous Mobile Platforms," *in Proc. of Design Automation and Test in Europe Conference*, March 2015.

[2] Ganapati Bhat, Suat Gumussoy, and Umit Y. Ogras. "Power-Temperature Stability and Safety Analysis for Multiprocessor Systems," *in ACM Tran. on Embedded Comp. Sys. (ESWEEK Special Issue)*, October 2017.